Fundamentals of Multichannelstructural Analysis of Electrical Signals

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***Abstract*— the existing structural uncertainty of the alarm process signal, which consists in the unknown dimension of the model and the uncertainty of the type of the process terms, requires the use of special methods and models of signal recognition that can work under conditions of a priori uncertainty. The resolution of the structural model is affected by the sampling frequency of the input signal, the competition of the components of the effective core filter, the intermodel decimation of the signal samples, the decimation of the residual samples, and the order of the initial filter. As the filter order increases, the signal processing window increases, so an unjustified increase in the order of the adaptive filter is undesirable. This report discusses a new approach to adaptive structural analysis based on a multi-channel adaptive filter. The advantages of multi-channel structures are the possibility of a different step within the model decimation in the filters.**

Keywords—adaptive structural analysis, Prony method, signal processing, canonical filter, core filter, noise filter, composite filters, relay protection

I. INTRODUCTION

In the information environment of a digital substation, current and voltage signals exist in the form of Sampled Values Stream data, the properties of which cannot be predetermined by algorithms for the digital processing of relay protection devices. In this regard, the characteristics of methods for recognizing the structure of a digital signal are determined only by the substantive properties of structural models and methods for controlling their dimension [1].

It is known [2] that one of the fundamental properties that determine the characteristics of the classical structural model is the cardinal dependence of its recognition ability on the noise filter potential. At the same time, the noise filter for the classical adaptive structural model is only a formal concept and can be allocated as a separate functional block of the structural model only after the model complete is tuning. Therefore, the classical structural model, concentrating all the information about the characteristic parameters of the signal in its characteristic polynomial, cannot represent the signal structure in the form of models with distributed parts. This limits the potential of the classical structural model, reducing its performance.

Quite recently, it was discovered [3] that further improvement of methods for recognizing the signal structure is associated with the use of structural models with distributed parts. This is the topic of this report. It discusses the basics of a new method of multichannel adaptive structural analysis using distributed structural models of the electrical signal.

In contrast to the classical model, the multichannel structural model reduces the internal competition of the canonical filters of the effective core, which reduces the overall order of the filter.

II. THE BEGINNING OF THE THEORY OF ADAPTIVE MODELS

The origins of the theory of adaptive models go back to the Prony method. In 1795, Prony used a method based on fitting an exponential model to measurements at regular intervals to interpolate the data of his experiments on gases. In the original article by Prony [4], the method is described for the case of an exact fit of the exponents to the available measurements, while the number of samples used is equal to the number of exponents. The modern presentation of the Prony method for the case when the number of samples far exceeds the number of exponents, as well as the development for the case of complex exponents, is presented in many scientific papers; the most famous of them is the work [5].

The Prony method in its original form is of little use for practical application since the additive noise in the signal leads to a significant variance in the estimates of the exponential arguments. But Prony discovered the main property of his model, which is that if the terms of the exponential model are the basis of the eigenfunctions of a certain difference equation, then the interpolation function satisfies this differential equation. It is this property of the model that is the essence of the Prony method since it justifies the connection between the roots of the characteristic equation with the arguments of the exponents. However, if Prony had been familiar with operational calculus, he would have easily established this connection using the Laplace transform [2].

When considering the Prony method, it should be borne in mind that it was used in its original form to approximate a sequence of data and then calculate (interpolate) the approximated function at intermediate points. The method was not intended to determine the structure of the signal at all.

The task of recognizing the signal structure of electrical systems has become relevant with the increase in computing resources of digital relay protection and emergency control systems. At first, when relay protection terminals could not yet provide a developed computing environment, algorithms of digital protections were based mainly on simple methods of determining the parameters of the fundamental harmonic of the steady-state and transient process of the electric network. Methods were used to calculate the amplitude and phase of the first harmonic of the electric quantity, based on the assumption that they have a sinusoidal form.

Then algorithms were developed for non-sinusoidal signals as well. The desire to estimate the parameters of the fundamental harmonic, resorting to the wealth of possibilities offered by digital filters, led to the widespread introduction of the relay protection technique of orthogonal component filters, based most often on the Fourier transform.

As relay protection terminals were improved, the first experiments were undertaken to optimally estimate the fundamental harmonic of the transient mode signal of an electrical system. The least-squares method was used with a rather computationally expensive singular decomposition as the solution tool.

Most importantly, all of the proposed algorithms were designed for evaluating the fundamental harmonic, and without structural analysis of the signal (determination of the signal structure). Therefore, they cannot be effectively used in modern systems of adaptive control of electric networks, adaptive relay protection, or fault location systems.

Furthermore, the application of structural analysis makes it possible to expand the functionalities of centralized grid control systems, when to avoid overloading the communication lines and the computing resources of high-level systems, it is necessary to transmit the information on the processes in the network recorded in a remote point in a compressed form by conducting a structural compression of oscillograms. Along with this structural analysis allows building a hardware-software complex of recognition of the weak signal component on the background of the dominant signal components, which are, in fact, insurmountable interference to it.

III. Canonical Component Filter as the Main Element of Distributed Structural Models

Structural analysis of the electrical signal representation of the signal assumes a sum of its components [9]. A filter that rejects the signal component is called canonical.

So, the aperiodic component

$$x(k) = e^{-\alpha T_s k}$$

will be rejected by the canonical filter

$$e(k) = x(k) + a_1 x(k-1),\ \ a_1 = -e^{-\alpha T_s}, \qquad (1)$$

a decaying oscillation

$$x(k) = e^{-\alpha T_s k}\sin(\omega T_s k + \psi) -$$

by the canonical filter

$$e(k) = x(k) + a_1 x(k-1) + a_2 x(k-2),$$
$$a_1 = -2e^{-\alpha T_s}\cos(\omega T_s),\ \ a_2 = e^{-2\alpha T_s}, \qquad (2)$$

where T_s – sampling period, k – sample number.

Therefore, theoretically, the structural model is a cascade of filters that reject the signal components [3, 4].

IV. Classical Adaptive Filter as a Structural Model

The classical structural model is an adaptive filter tuned to the signal reject, the part of the characteristic roots of which are aligned to the signal components. The other part of its roots is not associated with the signal but may contain roots that, in principle, cannot be separated from the signal roots. [1]. Together, they form the roots set of the model effective core. The rest of the roots that are inconsistent with the signal form a noise filter. Therefore, the classical structural model tuned to the signal – the effective structural model [3] – can be represented by an effective core filter (ECF) and a noise filter (NF) (Fig. 1). In turn, the effective core filter will consist of a cascade of canonical filters defined by roots related to the signal roots set (Fig. 2).

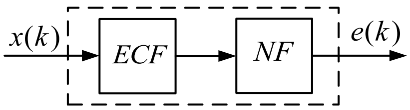

Fig. 1. Basic components of a classical adaptive structural model

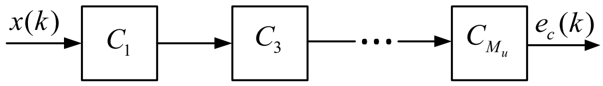

Fig. 2. Structure of the effective core filter: C_i – canonical filter of the i-nd component, M_u – number of components

The division of the classical structural model into an effective core filter and a noise filter is conditional, although it is performed according to well-founded rules [1]. Such a representation of the model is important purely methodically for forming a component model of a signal and explaining the fundamental properties of adaptive structural models [2]. Unfortunately, the classical model cannot take advantage of such a separation, since its tuning is carried out taking into account all the coefficients, even if the model is represented by a cascade of separate filters. It is this circumstance that limits the maximum performance of the classical adaptive structural model.

Let's show the application of rules of division of the classical structural model into an effective kernel filter and a noise filter on the example of decomposition of the structural model and construction of the component model of the signal of the emergency process, samples of which are taken from work [2].

The order of the useful signal is 5 (the signal contains the first and third harmonics and aperiodic components), i.e. $M_S = 5$. The adaptive structural model order took more than the order of the useful signal to illustrate the application of the division rules of the classical model.

During adjustment and decomposition of the adaptive filter tuned to the signal, the interactive adaptive structural analysis environment described in [2] was used (Table 1).

TABLE I. Parameters of the Structural Model

Parameter	Value
Sampling interval, T_s	1/1200c
Signal length, L	39
The structural model order, M	19
Model within decimation, ν	1
The structural model tuning method	TLS - the solution with the minimum norm (Total Least Squares Method)

The characteristics of the adaptive structural model are presented in Table 2. The frequency estimates and decay coefficients of the components of the recognized signal are determined by the root agreement equation. The assignment of roots to one or another region is performed according to the rules of root separation described in [1].

TABLE II. DECOMPOSITION OF THE ADAPTIVE STRUCTURAL MODEL

№	The adaptive model root $\underline{\zeta}_i$	Root modulus $\lvert\underline{\zeta}_i\rvert$	Relative freq. $\Omega = \omega T_s$, degree	Freq. $\hat{f}_i$, Hz	Decay coef. $\hat{\alpha}_i$, s^{-1}
1 2	0,9654± j0,26	0,9998	±15,1	50,24	0,02
3 4	0,7080± j0,7085	1,0016	±45,0	150,06	1,89
5	0,9406	0,9406	0	0	73,5
6 7	0,3511± j0,7074	0,7898	±61,6	212,02	285,7
8 9	0,2107± j0,7927	0,8202	±75,1	250,38	238,1
10 11	–0,0754± j0,8393	0,8202	±95,1	317,11	204,1
12 13	–0,3432± j0,7703	0,8433	±114,0	380,04	204,1
14 15	–0,5765± j0,6171	0,8445	±133,1	443,51	204,1
16 17	–0,7414± j0,3982	0,8419	±151,8	505,91	208,3
18 19	–0,8300± j0,1347	0,8409	±170,8	569,28	208,8

The first four roots belong to the harmonic roots. They are consistent with the basic and third harmonics. It could be assumed that the roots with numbers 8 and 9 of the adaptive filter are associated with a harmonic frequency of 250 Hz, but because of the significant decay coefficient (238.1 s^{1}) and high relative frequency (75.1 degree), they refer to roots uncoordinated with the signal. The remaining damped oscillations of the component model are also defined by the rules as unconnected with the signal. No causal roots appeared among the roots of the adaptive structural model.

Thus, the signal model decomposition rules determined the effective kernel polynomial for the first five roots:

$$P_{M_c}(\underline{\zeta}) = 1 - 4.285\underline{\zeta}^{1} + 7.876\underline{\zeta}^{2} - \\ -7.794\underline{\zeta}^{3} + 4.146\underline{\zeta}^{4} - 0.9406\underline{\zeta}^{5} \quad (3)$$

by isolating it from the polynomial of the general adaptive structural model. Noise filter polynomial:

$$P_{M-M_c}(\underline{\zeta}) = 1 + 4.0101\underline{\zeta}^{-1} + 9.1621\underline{\zeta}^{-2} + \\ +15.4167\underline{\zeta}^{-3} + 20.9870\underline{\zeta}^{-4} + 24.1714\underline{\zeta}^{-5} + \\ +24.1071\underline{\zeta}^{-6} + 21.0643\underline{\zeta}^{-7} + 16.1904\underline{\zeta}^{-8} + \\ +10.9259\underline{\zeta}^{-9} + 6.4193\underline{\zeta}^{-10} + 3.2239\underline{\zeta}^{-11} + \\ +1.3322\underline{\zeta}^{-12} + 0.4152\underline{\zeta}^{-13} + 0.0757\underline{\zeta}^{-14}. \quad (4)$$

The amplitude-frequency characteristics of the effective core and noise filters are shown in Figure 3.

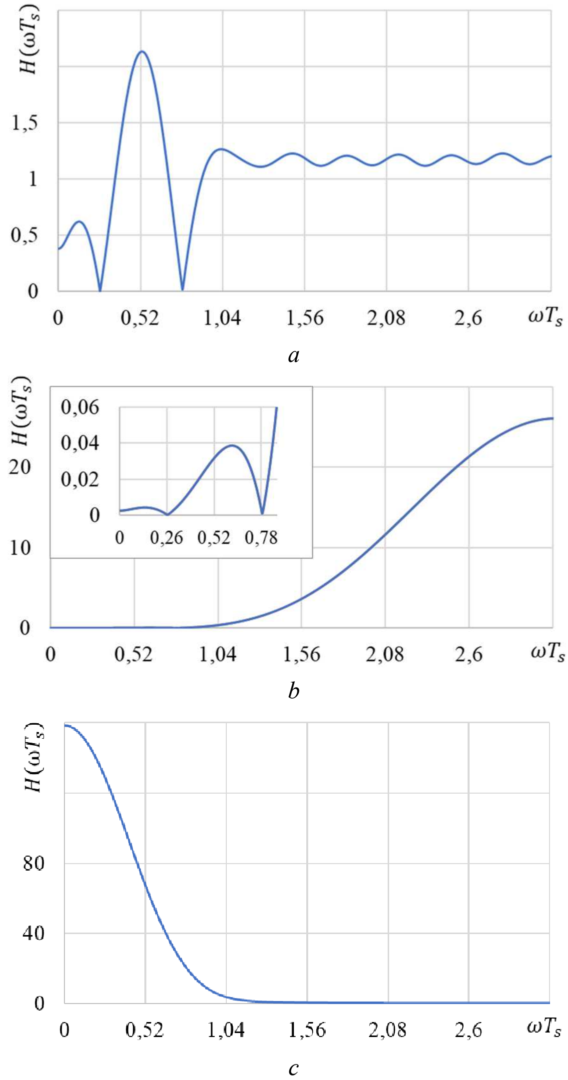

Fig. 3. The amplitude-frequency response of the general adaptive filter (a) effective core filter (b) and noise filter (c)

V. COMPOSITE FILTER AS A COMPONENT MODEL

The composite model of the term is formed as a filter created from the set of roots of the structural model of the signal after excluding the root of the recognized term [2]. The composite filter will block all components of the signal, except for the component whose root is missing in its characteristic polynomial. Therefore, the signal at the output of the composite filter will be proportional to the recognized term.

In general, a composite filter is a cascade of canonical filters of signal components (except the canonical filter of the recognized component) and a noise filter. The composite filter of a component is convenient as a tool for evaluating the capabilities of an adaptive filter when recognizing a component and, in addition, characterizes the influence of canonical filters of other components and a noise filter on this component.

Figure 4 shows the operation of the composite filters of the model formed in the form of a cascade of filters (1) and (2) for the signal

$$x(k) = e^{-0,06k} + e^{-0,03k}\cos\left(\frac{\pi}{12}k\right) \qquad (5)$$

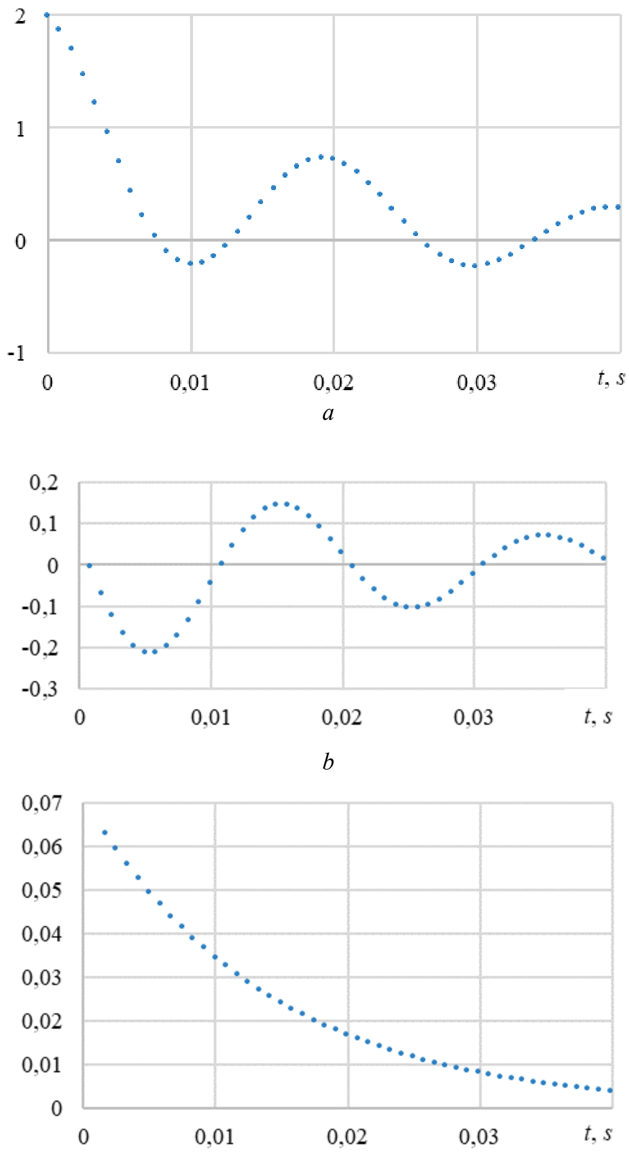

Fig. 4. Isolation of the damped oscillation (b) and the aperiodic component (c) by composite filters obtained from the model in the form of filters (1) and (2) of the original signal (a)

The reaction of composite filters vividly illustrates the competition of canonical filters (1) and (2): each of them significantly weakens the component blocked by the other.

VI. Residual Signal Filter as a Basis for Multichannel Structural Analysis

The residual signal filter is a filter that is tuned to the output signal of a cascade of canonical filters [7]. Interestingly, the more precisely the canonical filter is adjusted to the barrier of its component, the better conditions are created for tuning the residual signal filter to components that are not provided for in the cascade of canonical filters. The opposite statement is also true, since in this case, the residual signal filter behaves to canonical filters as a composite filter, selectively amplifying their components.

This property of the elements of a multichannel adaptive filter creates a positive feedback effect, due to which the convergence of the tuning procedure of individual parts of the distributed structure of the adaptive filter increases. As a rule, the number of iterations σ does not exceed 5.

The uniqueness of the residual signal filter also lies in the fact that it takes on the task of blocking the components that remain free after the canonical filters work, and forms a noise filter in its structure, thereby creating a solid basis for recognizing the entire signal structure.

VII. Multichannel Adaptive Filter as the Basis of Multichannel Structural Analysis

The multichannel adaptive filter (Fig. 5) is a set of channels intended for tuning the canonical filters of components C_i and the channel for tuning the residual signal filter F_n [5]. The number of canonical filters in a multichannel system is set based on a priori information about the signal structure or following the requirement that the component must be determined directly without analyzing the roots of the characteristic equation of the adaptive filter. Each channel has its solver, which forms either an estimate of the coefficients $\mathbf{a}_i^\sigma$ of the canonical filter C_i or $\mathbf{a}_n^\sigma$ the residual signal filter at the current stage σ. The methods used by the solver to configure filters can be different [3, 8].

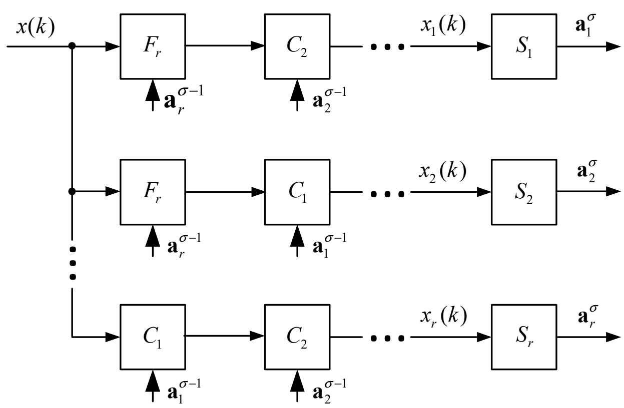

Fig. 5. Structure of a multichannel adaptive filter

An important advantage of the multichannel adaptive filter is the reduction of the general order of the model due to the exclusion of internal competition of canonical component filters by using the distributed structure of the adaptive filter [9,10]. Moreover, the adaptive filter acquires this opportunity precisely because of the multi-channel and iterative filter setup procedure. Each iteration enhances the role of the residual signal filter, which is a multi-channel filter, plays the role of a composite model of components recognized by the canonical filters preceding it [11]. Therefore, all the properties of the composite model of the term are inherent in the residual signal filter.

Let us show the configuration of the multi-channel adaptive filter by the example of building the component model of the signal considered in paragraph IV.

The order of the useful signal is 5 (the signal contains first and third harmonics and aperiodic components), i.e. the order of the multichannel adaptive structural model is 9.

The filter coefficients of the effective core of the classical adaptive filter and multichannel filter are identical, so their amplitude-frequency characteristics coincide (fig. 3b). The amplitude-frequency response of the multichannel noise filter is shown in fig. 6.

Thus, the use of the distributed adaptive filter structure made it possible to reduce the overall order of the model, due to the elimination of the internal competition of the canonical component filters (from 14 to 9 for the signal in question).

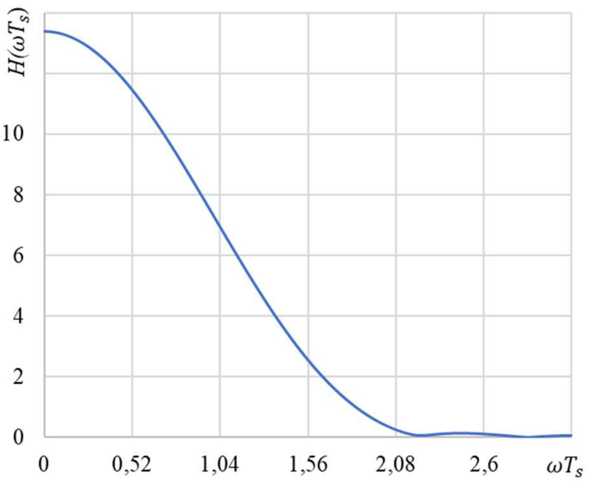

Fig. 6. Noise filter amplitude-frequency response of a multichannel system

VIII. Conclusion

The method of multichannel structural analysis proposed by the authors is fundamentally different from the classical structural analysis, because it changes the ideology of the adaptive structural model tuning, smoothing the effect of competition of the effective core filter components.

A multi-channel adaptive filter, comprising the channels for configuring canonical filters and the residual signal filter, forms a distributed system for recognizing the signal structure. The perfection of the structural model created by him is ensured by the emergence of positive feedback between different parts of the multichannel system, due to which the components recognized by canonical filters do not participate in the competitive environment of recognition of unknown components by the residual signal filter. It is this property of a multichannel adaptive filter that creates favorable conditions for recognizing the signal structure on a small number of samples by a low-order filter, increasing the speed of relay protection using distributed structural models.

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